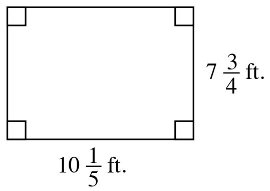
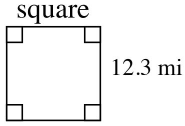
6-110. Find the area and perimeter of the following figures.

a.

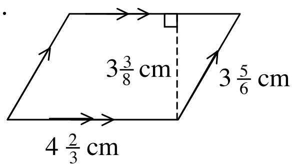


b.

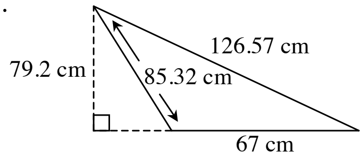
square



c.



d.



6-111. Graph the following points on a coordinate grid: $(1,1)$, $(4,1)$, and $(3,4)$.

Connect the points, then translate the points three units right and three units up. What are the coordinates of the vertices of the new triangle?

6-112. Find the value of the expression $2x+6$ for the given values of x.

a. $x=6$ b. $x=-2$ c. $x=0$ d. $x=5$

6.2.5 How can I model it?

Writing and Solving Equations

Engineers investigate practical problems to improve people's quality of life. To investigate solutions to problems they often build models. These models can take various forms. For example, a structural engineer designing a bridge might build a small replica of the bridge. Civil engineers studying the traffic patterns in a city might create equations that model traffic flows into and out of a city at different times.

In this lesson you will be building equations to model and solve problems based on known information. As you work today keep the following questions in mind:

What does x represent in the equation?

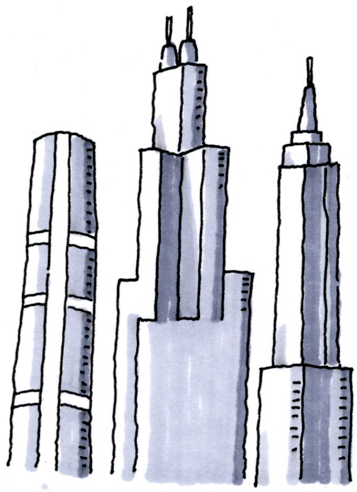
How does the equation show the same information as the problem?

Have I answered the question?

6-113. Today your team will be responsible for solving a problem and sharing your solution with the class. It is important that your poster communicates your thinking and **reasoning** so that people who look at your poster understand how you solved the problem. Your poster should include:

- Connections between the words in the problem and the relationships in your chart and/or equation. Connections can be made with arrows, colors, symbols, and labels.
- Variables that are defined completely.
- An equation to represent the problem.
- Your solution to the problem.
- The answer declared in a sentence.

Begin by solving one of the problems below and writing an equation. Make sure to define the variable you use and answer the question(s) being asked. Using the 5-D Process, including numerical trials, may be helpful.



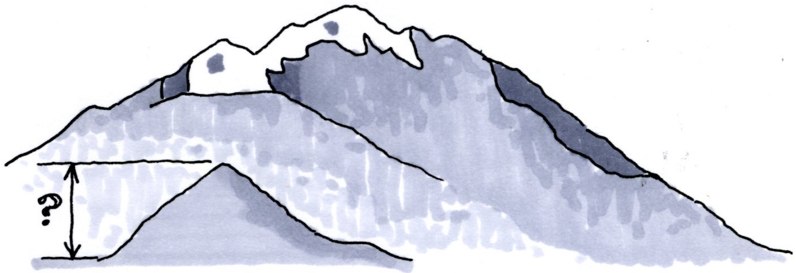
a. Hong Kong's tallest building, Two International Finance Center, is 88 stories tall. The former Sears Tower in Chicago is eight stories taller than the Empire State Building in New York City. If all of the buildings were stacked on top of each other, the combined heights would have 300 stories. How many stories does the Sears Tower have?

b. Have you ever driven or walked across a suspension bridge? There are many suspension bridges in the world of different lengths that allow people to travel across rivers, bays and lakes.

The Mackinac Bridge in Michigan is 1158 meters long. The Tsing Ma Bridge in Hong Kong is 97 meters longer than the Golden Gate Bridge in California. Together, all three bridges have a length of 3815 meters. How long is the Tsing Ma Bridge?

Problem continues on next page. →

6-113. *Problem continued from previous page.*

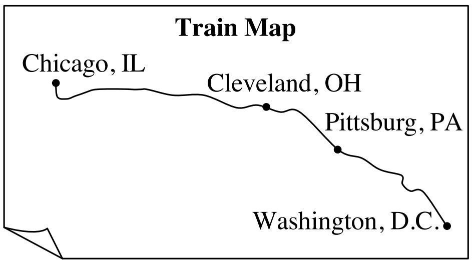


c. Elevations found in the United States range from California's Death Valley at 282 feet below sea level to Alaska's Mount McKinley at 20,320 feet above sea level.

The highest elevation in Delaware is 106 feet higher than the highest elevation in Florida. Louisiana's highest elevation is 190 feet higher than Florida's highest elevation. If you climbed from sea level to the highest points in Delaware, Florida, and Louisiana, you would only climb 1331 feet. How high is the highest elevation in each of the three states?

d. Most states in the United States are divided into counties. Some counties are very large and some are very small, and different states have different numbers of counties. Pennsylvania has five less than twice as many counties as Oregon. Florida has one less county than Pennsylvania. All together, the three states have 169 counties. How many counties does Florida have?

e. A train from Washington, D.C. to Chicago first stops in Pittsburg and then in Cleveland. The distance from Washington, D.C. to Pittsburg is 30 miles less than twice the distance from Pittsburg to Cleveland. The distance from Cleveland to Chicago is 220 miles more than the distance between Pittsburg and Cleveland. If the entire train ride is 710 miles, how far is the train ride from Cleveland to Chicago?



Review & Preview

6-114. On graph paper, graph the rule $y = 4x - 6$. Determine the value of y when $x = \frac{1}{2}$.

6-115. Solve the following equations using any method. Show your work and check your solution.

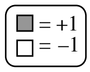
a. $2x + 16 = 5x + 4$ b. $3x - 5 = 2x + 14$ c. $5x - 5 = x + 15$

6-116. Janet lit a 12-inch candle. She noticed that it was getting an inch shorter every 30 minutes.

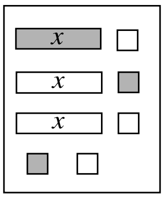
a. Is the correlation between the time the candle is lit and the height of the candle positive or negative?

b. In how many hours will the candle burn out? Support your answer with a **reason**.

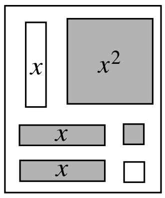
6-117. Write the expression as shown on the expression mats, then simplify by making zeros and combining like terms.



a.

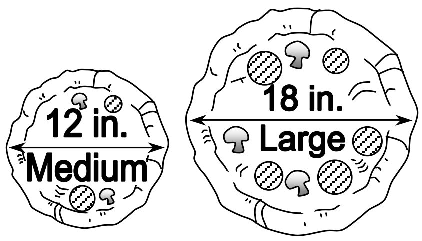


b.



6-118. Whole pizzas are described by the length of their diameters. For example, a 12-inch pizza has a diameter of 12 inches.

Stan the Pizza Man told a customer that a 12" medium pizza costs \$10 and an 18" large pizza costs \$16. Which pizza is the better deal for the customer (that is, which one costs less per square inch)? You may use a calculator, but you must show your work.



6.2.6 Is there always a solution?

Cases With Infinite or No Solutions

Are all equations solvable? Are all solutions a single number? Think about this: Annika was born first and her brother William was born 4 years later. How old will William be when Annika is twice his age? How old will William be when Annika is exactly the same as his age?

In this lesson you will continue to practice your **strategies** of combining like terms, removing zeros and balancing to simplify and compare two expressions, but you will encounter unusual situations where the solution may be unexpected. As you work today, focus with your team on these questions:

What if both sides are not equal?

Are there many values of x that will make the expressions equal?

Is there always a solution?

6-119. Many students believe that every equation has only one solution. However, in the introduction to this lesson you might have noticed that if Annika was four years older than her brother, William, they could never be the same age. There are sometimes situations that have "one solution," or, "no solution," or "all numbers" as solutions.

For each of the following equations, **reason** with your team to decide if there is "One solution," "No solution," or "All numbers are solutions." If there is a single number solution, write it down. If you are not sure how many solutions there are, have each member of your team try a number to see if you can find a value that makes the equation work.

a. $x = x$

b. $x + 1 = x$

c. $x = 2x$

d. $x + x = 2 + x$

e. $x + x = x - x$

f. $x + x = 2x$

g. $x \cdot x = x^2$

h. $x - 1 = x$

6-120. Use the 5-D Process to write an equation for the problem below. Then, answer the question.

Kelly is 6 years younger than her twin brothers Bailey and Larry. How old will Kelly be when the sum of their ages is 12 more than three times Kelly's?

6-121. SPECIAL CASES – Part One

Use the equation $8 + x + (-5) = (-4) + x + 7$ to complete parts (a) through (c).

a. Build the equation on your Equation Mat and simplify it as much as possible. Record your steps and what you see when you have simplified the equation fully. Draw a picture of your final mat.

b. Have each member of your team test a different value for x in the original equation, such as $x = 0$, $x = 1$, $x = -5$, $x = 10$, etc. What happens in each case?

c. Are there any solutions to this equation? If so, how many?

6-122. SPECIAL CASES – Part Two

Use the equation $x + x + 2 = 2x$ to complete parts (a) through (c).

a. Build the equation on your Equation Mat and simplify it as much as possible. Record your steps and what you see when you have simplified the equation fully. Draw a picture of your final mat.

b. Have each member of your team test a different value for x in the equation, such as $x = 0$, $x = 1$, $x = -5$, $x = 10$, etc. What happens? Is there a pattern to the results you get from the equation?

c. Did you find any values for x that satisfied the equation in part (a)? When there is an imbalance of units left on the mat (such as $2 = 0$), what does this mean? Is $x = 0$ a solution to the equation?

6-123. Keeping these special cases in mind, continue to develop your equation solving **strategies** by visualizing and solving each equation below. Remember to build each equation on your mat, simplify as much as possible, and solve for x. Identify whether one number is the solution, any number is the solution, or there is no solution. Record your steps.

a. $-x+2=4$

b. $-3+x=2(x+3)$

c. $5x+3+(-x)=2x+1+2x+3$

d. $3x+7+(-x)+-2=2x+5$

e. $4+-3x=2$

f. $3x+3=4+x+(-1)$

METHODS AND MEANINGS

MATH NOTES

Solutions to an Equation With One Variable

A **solution** to an equation gives the value(s) of the variable that makes the equation true. For example, when 5 is substituted for x in the equation at right, both sides of the equation are equal. So $x=5$ is a solution to this equation. Some equations have several solutions, such as $x^2=25$, where $x=5$ or -5.

$$4x-1=2x+9$$
$$4(5)-1=2(5)+9$$
$$19=19$$

Equations may also have no solution or an infinite (unlimited) number of solutions.

Notice that no matter what the value of x is, the left side of the first equation will never equal the right side. Therefore, we say that $x+2=x+3$ has **no solution**.

Equation with no solution:

$$x+2=x+3$$
$$2\neq 3$$

However, in the equation $x-2=x-2$, no matter what value x has, the equation will always be true. So all numbers can make $x-2=x-2$ true. Therefore, we say the solution for the equation $x-2=x-2$ is **all numbers**.

Equation with infinitely many solutions:

$$x-2=x-2$$

Review & Preview

6-124. Copy and simplify the following expressions by combining like terms, making zeros, and using the Distributive Property. Using algebra tiles may be helpful.

a. $(-1)+4x+2+2x+x$

b. $-8x+4+(-3)+10x$

c. $(-4)+1x^2+3x+4$

d. $2(3x-2)$

6-125. Simplify and solve each equation below for x. Show your work and record your final answer.

a. $24=3x+3$

b. $12+x=2x-2$

6-126. Consider the following table.

x	–3	0.5	0	2	4
y	9	2	3	–1	–5

a. What is the rule for the table?

b. Explain your **strategy** for finding the rule for this table.

6-127. Show the "check" for each of these problems and write whether the solution is correct or incorrect.

a. For $3x+2=x-2$, does $x=0$?

b. For $3(x-2)=30+x-2-x+2$, does $x=12$?

6-128. Some steps in solving an equation are more efficient than others. Complete parts (a) through (d) to determine the most efficient first step to solve the equation $34=5x-21$.

a. If both sides of the equation were divided by 5, then the equation would be $\frac{34}{5}=x-\frac{21}{5}$. Does this make the problem simpler? Why or why not?

b. If we subtract 34 from both sides, the equation becomes $0=5x-55$. Does this make the equation simpler to solve? Why or why not?

c. If we add 21 to both sides, the equation becomes $55=5x$. Does this suggestion make this a problem you can solve more easily? Why or why not?

d. All three suggestions are legal moves, but which method will lead to the most efficient solution? Why?

6.2.7 Which method should I use?

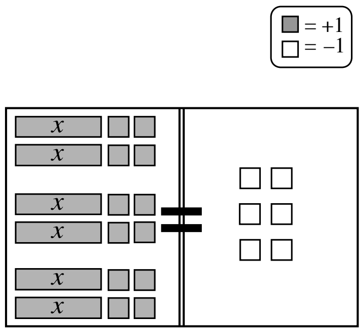
Choosing a Solving Strategy

The 5-D Process and algebra tiles are useful tools for solving problems. Today you will practice writing equations from word problems and solving them using any of the tools you know. We are developing an efficient set of tools to solve any word problem. Having a variety of methods will allow you to choose the one that makes sense to you and ultimately makes you a more powerful mathematician.

6-129. Nick tried to use the symbolic **visualization** shortcut for $3x - 6 = 27$, but may have made a mistake. His work is shown at right. If he did, on which step did he first make a mistake and what was his mistake? If he did not make a mistake, check his solution and write "all correct."

$$\begin{aligned} 3x - 6 &= 27 \\ +6 &= +6 \\ \hline \frac{3x}{3} &= \frac{33}{3} \\ x &= 11 \end{aligned}$$

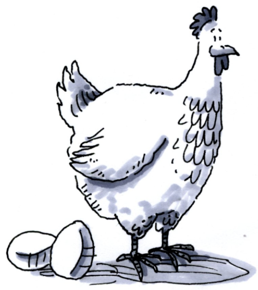
6-130. Nick represented the equation $3(2x + 4) = -6$ on the Equation Mat at right.



a. Choose a strategy to solve for x. You may continue to use algebra tiles or may use numbers and variables.

b. Check your answer. If your answer does not make the equation true, try solving the equation using a different strategy.

6-131. Read the problem below, then answer the questions in parts (a) and (b).

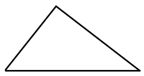


Cisco and Misty need to construct a chicken coop for their famous egg-laying hens. The hens need at least 108 square feet of living space. The space available allows Cisco and Misty to make the length of the coop 3 feet longer than the width and to create exactly 108 square feet of area. What are the dimensions of the coop?

a. Laura tried to use an Equation Mat for this problem but got stuck and decided to use a different **strategy**. Why do you think she decided not to use the Equation Mat?

b. Choose a different method, such as the 5-D Process or writing and solving an equation to solve this problem. Even if you solve an equation and do not use the chart from the 5-D Process, you still need to define how you are using variables and remember to write a sentence answer. Check your answer.

6-132. Here is a problem started in a 5-D table.

Describe/Draw: 

We want to find the side lengths so the perimeter is 35.

	Define			**Do**	**Decide**
	Side 1	Side 2	Side 3	Perimeter	Target: 35 ft
Trial 1	5	$5+2=7$	$7+4=11$	$5+7+11=23$	23 *Too small*
	x	$x+2$	$(x+2)+4$	$x+(x+2)+(x+2+4)=35$	

a. Write the word problem that could have accompanied this 5-D table.

b. What is your preferred method to solve this problem: algebra tiles, an equation, or the 5-D Process?

c. Decide on a method to solve this problem, use your method to find your answer, and write a Declare statement for your answer.

6-133. Apiologists (scientists who study bees) have found that the number and types of bees in a hive is based on the amount of nectar and pollen available. Within a hive there are three types of bees that help the queen. They are workers, drones, and nurses.

In a recent study of a hive it was found that, not including the queen, there were a total of 4,109 bees. There were thirty-three more nurses than drones. The number of workers was twelve more than six times the number of drones. How many of each type of bee was in the hive?

Choose a method to solve this problem such as the 5-D Process or writing and solving an equation. Even if you solve an equation and do not use the chart from the 5-D Process, you still need to define how you are using variables and remember to Declare your answer in a sentence.

6-134. Use any method to solve the following equations. Show your work.

a. $3x+4=-5$

b. $3(x+4)=-3$

c. $3(x+4)=x+2(x+6)$

d. $3x+4=3x-4$

Review & Preview

6-135. Beth is filling a small backyard pool with a garden hose. The pool holds 300 gallons of water. After 15 minutes the pool is about one-fourth full.

a. Assuming that the water is flowing at a constant rate, about how much water is going into the pool each minute?

b. About how long will it take to fill the pool?

6-136. One way of thinking about solving equations is to work to get the variable terms on one side of the equation and the constants on the other side. Consider the equation $71 = 9x - 37$.

a. As a first step you could subtract 71 from both sides, or divide both sides by 9, or add 37 to both sides of the equation. Does one of these steps get all of the variable terms on one side of the equation and the constants on the other?

b. Solve $71 = 9x - 37$ for x. Show your steps.

6-137. Make a table and a graph for the following rule: $y = 3x - 2$.

a. Explain how you know that there are more points than just the ones shown on your graph.

b. Where would the additional points go on the graph?

6-138. For each equation below, solve for x. Sometimes the easiest **strategy** is to use mental math.

a. $x - \frac{2}{3} = \frac{1}{3}$ b. $4x = 6$ c. $x + 4.6 = 12.96$ d. $\frac{x}{7} = \frac{3}{7}$

6-139. Explain what the following graphs would look like. You may write your answer in words or sketch a graph.

a. Scatterplot with a negative correlation.

b. Scatterplot with no correlation.

c. Scatterplot with a positive correlation.

Chapter 6 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics with which you need more help. Look for connections between ideas as well as connections with material you learned previously.

① SUMMARIZING MY UNDERSTANDING

In this chapter you have used algebra tiles and an equation mat as tools for solving equations, and you have represented your solution steps on an equation mat and with algebraic symbols. Today you will use what you have learned about equations in this chapter to show connections between all of these methods. To start, consider this problem:

Jamee is working to solve an equation. She did the work shown below. With your team, answer the following questions:

Jamee's work: Original problem: $3(2x-4)=2(2x+5)$

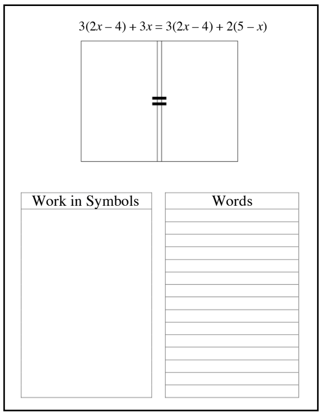
Step 1: $6x-12=4x+10$

Step 2: $2x=-2$

- Explain what Jamee did at each step.
- What is her solution?
- Was her solution correct? **Justify** your answer. If it was not, find her error and the correct answer.

Problem continues on next page. →

① *Problem continued from previous page.*



Obtain a Chapter 6 Closure Resource Page (shown at right) from your teacher. Follow the directions below to demonstrate your understanding of solving equations with an equation mat, algebraically (with numbers and symbols), and in words.

Part 1: Sketch the equation on the mat on the resource page. You might also want to build it with algebra tiles.

Part 2: Complete each step to solve the equation. Represent each step on the mat, in symbols, and in words. As you work, ask questions to clarify your thinking and understanding. Make sure you can give reasons for each step.

Part 3: Color code the matching steps in each representation. For example, if your second step is to combine like terms, label this step with green in the symbols, on the mat, and in words. Use a new color to code each step.

② WHAT HAVE I LEARNED?

Working the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with.

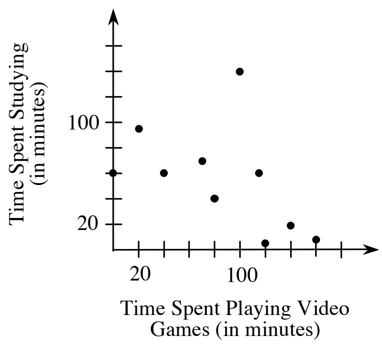
Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also directs you to additional help and other similar practice problems.

CL 6-140. Ruthie did a survey among her classmates comparing the time spent playing video games to the time spent studying. The scatterplot of her data is shown at right.

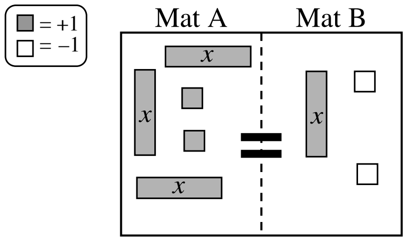
a. What correlation can you make from her data?

b. Use an ordered pair (x, y) to identify any outliers.

Playing Video Games



CL 6-141. Consider the equation mat at right.



a. Write the original equation represented.

b. Simplify as needed. Record all steps of your work. What value of x will make the two sides equal?

c. Check your solution.

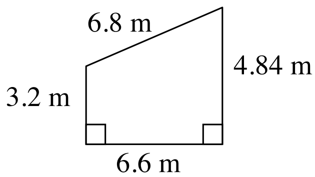
CL 6-142. Make a table with at least 6 entries and a graph for the following rule: $y = 3x - 5$.

a. If you added four more entries to your table and plotted the points, where would the points be on the graph?

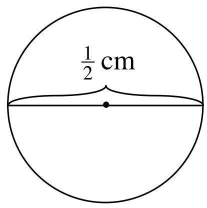
b. Is there a limit to the number of points you could put in your table? Explain how you know.

CL 6-143. Find the area and perimeter or circumference of the following figures. Approximate π as 3.14.

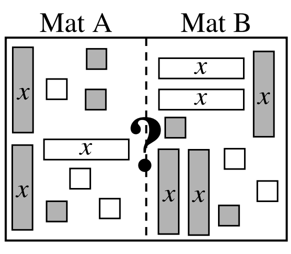
a.



b.



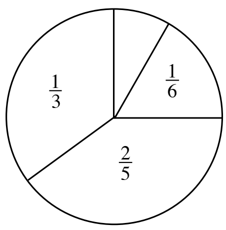
CL 6-144. Write the expressions for the expression mats on the right.



a. Simplify each mat to determine which side is greater.

b. If $x = 4$, would your answer to part (a) change? Explain.

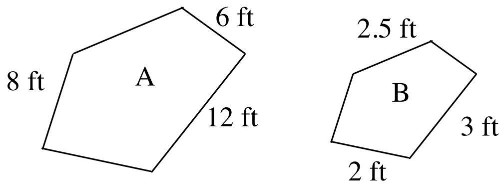
CL 6-145. Serena found the spinner at the right. Help her find the value of the missing portion.



CL 6-146. The shapes at right are similar.

a. What is the scale factor?

b. What are the lengths of the missing sides?

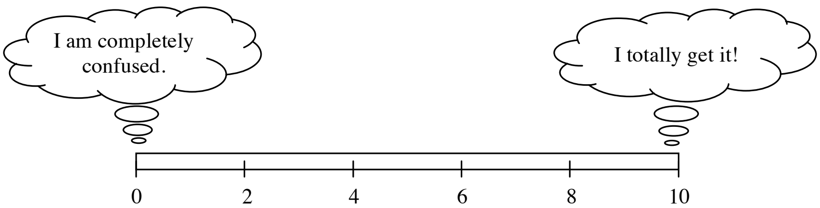


CL 6-147. Solve this problem using the 5-D Process or writing and solving an equation. No matter which you method you use, be sure to define your variable and write an equation to represent the relationship.

A rectangle has a perimeter of 30 inches. Its length is one less than three times its width. What are the length and width of the rectangle?

CL 6-148. For each of the problems above, do the following:

- Draw a bar or number line that represents 0 to 10.



- Color or shade in a portion of the bar that represents your level of understanding and comfort with completing that problem on your own.

If any of your bars are less than a 5, choose *one* of those problems and do one of the following tasks:

- Write two questions that you would like to ask about that problem.
- Brainstorm two things that you DO know about that type of problem.

If all of your bars are a 5 or above, choose one of those problems and do one of these tasks:

- Write two questions you might ask or hints you might give to a student who was stuck on the problem.
- Make a new problem that is similar and more challenging than that problem and solve it.

③ WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning – your teacher, your study team, your math book, and your Toolkit, to name only a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update your Toolkit to better reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Log, Math Notes, and the vocabulary used in this chapter. Math words that are new appear in bold in the text. Refer to the lists provided below and follow your teacher's instructions to revise your Toolkit, which will help make it useful for you as you complete this chapter and as you work in future chapters.

Learning Log Entries

- Lesson 6.1.3 – Correlations
- Lesson 6.1.5 – Scatterplot and Linear Graph Predictions
- Extension Activity – Human Data Points
- Lesson 6.2.3 – Solving Equations and Finding Solutions
- Lesson 6.2.4 – Using the 5-D Process to Write and Solve Equations

Math Notes

- Lesson 6.1.1 – Special Quadrilaterals
- Lesson 6.1.2 – Circle Graphs
- Lesson 6.1.4 – Correlation and Trend Lines
- Lesson 6.1.5 – Making a Complete Graph
- Lesson 6.1.6 – When Is a Point on the Graph of a Rule?
- Lesson 6.2.1 – Using an Equation Mat
- Lesson 6.2.3 – Checking a Solution
- Lesson 6.2.4 – Defining a Variable
- Lesson 6.2.6 – Solutions to an Equation With One Variable

Mathematical Vocabulary

The following is a list of vocabulary found in this chapter. Some of the words have been seen in the previous chapter. The words in bold are the words new to this chapter. Make sure that you are familiar with the terms below and know what they mean. For the words you do not know, refer to the glossary or index. You might also want to add these words to your Toolkit for a way to reference them in the future.

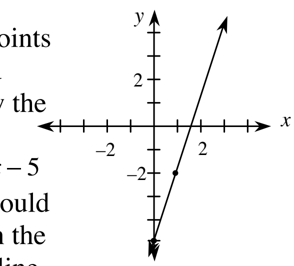
circle graph	**Equation Mat**	**negative correlation**
no correlation	**outlier**	**positive correlation**
solution	scatterplot	**trend line**
***x*-intercept**	***y*-intercept**	

Process Words

The list of words below are problem solving strategies and processes that you have been involved in throughout the course of this chapter. Make sure you know what it means to use each of them. If you are not sure, look through your book for problems when you were asked to think in the following ways.

choose a strategy	compare	describe
explain your reasoning	generalize	justify
predict	reason	simplify
solve	test your prediction	visualize

Answers and Support for Closure Activity #2
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 6-140.	a. Negative: when the time playing video games increases, the time spent studying decreases. b. $(100, 140)$	Lessons 6.1.2, 6.1.3, and 6.1.4 Math Notes box in Lesson 6.1.4 Learning Logs (problems 6-26 and 6-47)	Problems 6-14, 6-15, 6-24, 6-25, 6-26, and 6-49
CL 6-141.	a. $3x+2=x-2$ b. $x=-2$ c. $3(-2)+2=-2-2$ $-6+2=-4$ $-4=-4$	Lessons 6.2.1, 6.2.2, and 6.2.3 Math Notes boxes in Lessons 6.2.1, 6.2.3, and 6.2.6 Learning Log (problem 6-98)	Problems 6-75, 6-76, 6-77, 6-87, 6-89, 6-97, 6-99, 6-115, 6-125, 6-130, and 6-134
CL 6-142.	$\begin{array}{c\|c\|c\|c\|c\|c} x & 4 & -3 & 0.5 & 0 & 2 \\ \hline y & 7 & -14 & -3.5 & -5 & 1 \end{array}$ a. The points would follow the rule $y=3x-5$ and would fall on the same line.  b. There are an infinite number of points possible, because we can choose any value for x.	Lessons 6.1.2, through 6.1.6 Math Notes boxes in Lessons 6.1.5 and 6.1.6 Learning Log (problem 6-47)	Problems 6-37, 6-43, 6-48, 6-56, 6-59, 6-114, and 6-137
CL 6-143.	a. $A=26.532\ \text{m}^2$, $P=21.44$ m b. $A\approx 0.196\ \text{cm}^2$, $C\approx 1.571$ cm	Lessons 2.3.2, 2.3.5, 5.3.3, and 5.3.4 Math Notes box in Lesson 5.3.4 Learning Logs (problems 2-150 and 5-124)	Problems CL 2-163, CL 3-111, CL 4-116, 5-127, 5-129, and 6-11

Problem	Solution	Need Help?	More Practice
CL 6-144.	a. $x = x$, The mats are equal in value. b. No, the mats will be equal for any value of x.	Lessons 5.2.1, 5.2.2, and 6.2.1 through 6.2.6 Math Notes boxes in Lessons 5.1.2, 6.2.1, 6.2.3, and 6.2.6 Learning Log (problem 5-54)	Problems CL 5-134, CL 5-136, 6-38, 6-74, 6-75, 6-76, 6-78, and 6-117
CL 6-145.	$\frac{1}{10}$	Lessons 1.2.4 and 1.2.5 Math Notes box in Lesson 1.2.4 Learning Log (problem 1-107)	Problems CL 1-140, CL 2-160, CL 3-114, 4-45, and 5-12
CL 6-146.	a. Multiply by $\frac{1}{4}$ (or you can divide by 4 moving from shape A to shape B or multiply by 4 moving from shape B to shape A). b. On Shape A, the missing sides are 10 and 8 units; on shape B, the missing sides are 2 and 1.5 units.	Lessons 4.3.1 through 4.3.4 Math Notes boxes in Lessons 4.3.1 and 4.3.4 Learning Log (problem 4-76)	Problems CL 4-119, 5-10, 5-48, and 6-109
CL 6-147.	The length is 11 in. and the width is 4 in. If x = width, one possible equation would be $x+(3x-1)+x+(3x-1)=30$.	Lessons 3.2.3, 3.2.4, 3.2.5, 5.1.2, 5.1.3, 6.2.5, and 6.2.7 Math Notes boxes in Lessons 3.2.3 and 5.1.2 Learning Log (problems 5-32 and 6-107)	Problems CL 5-135, 6-97, 6-105, 6-113, 6-131, 6-132, and 6-133

Puzzle Investigator Problems

PI-11. GRAPHING MADNESS

On graph paper starting at (0, 0), carry out the following moves:

Move Number	Directions
1	Right 1 unit
2	Up 2 units
3	Left 3 units
4	Down 4 units
5	Right 5 units

Continue moving counter clockwise using this pattern, increasing the length 1 unit each move.

a. What patterns can you find in the figure on the graph? For example, find the coordinates of each point in the design. How are the coordinates changing? How is the quadrant where each point is located changing with the addition of each new point?

b. In which quadrant will the 79th move land? What will be the coordinates` of this point? Explain how you can find your answer without listing 79 moves.

c. For **any** move, name which quadrant it will be in and what its coordinates will be. Explain the method you are using.

PI-12. CONSECUTIVE SUMS

A consecutive sum is an addition sequence of consecutive whole numbers.

Examples: $2+3$

$8+9+10+11+12+13$

$7+8+9+10$

a. Write the first 35 counting numbers (1 through 35) with as many consecutive sums as possible. The number 15 can be written as $7+8$ or $4+5+6$ or $1+2+3+4+5$. Make sure you organize your work in order to find patterns.

b. Describe as many patterns as you find. For example, are there any numbers that have no consecutive sums? Which numbers can be written as a sum of two consecutive numbers? Which can be written as a sum of three consecutive numbers?

c. Ernie noticed a pattern with the numbers that can be written as a sum of three consecutive numbers. He wants to understand why the pattern works. He thinks it might help to represent the consecutive numbers as x, $x+1$, and $x+2$.

i. Why do these expressions represent consecutive numbers?

ii. Find the sum of x, $x+1$, and $x+2$. How does this sum help make sense of the pattern?